

Coherence-enhanced efficiency of feedback-driven quantum engines

Kay Brandner, Michael Bauer, Michael T. Schmid, Udo Seifert¹

E-mail: ¹useifert@theo2.physik.uni-stuttgart.de
II. Institut für Theoretische Physik, Universität Stuttgart, 70550 Stuttgart, Germany

Abstract. A genuine feature of projective quantum measurements is that they inevitably alter the mean energy of the observed system if the measured quantity does not commute with the Hamiltonian. Compared to the classical case, Jacobs proved that this additional energetic cost leads to a stronger bound on the work extractable after a single measurement from a system initially in thermal equilibrium [Phys. Rev. A 80, 012322 (2009)]. Here, we extend this bound to a large class of feedback-driven quantum engines operating periodically and in finite time. The bound thus implies a natural definition for the efficiency of information to work conversion in such devices. For a simple model consisting of a laser-driven two-level system, we maximize the efficiency with respect to the observable whose measurement is used to control the feedback operations. We find that the optimal observable typically does not commute with the Hamiltonian and hence would not be available in a classical two level system. This result reveals that periodic feedback engines operating in the quantum realm can exploit quantum coherences to enhance efficiency.

1. Introduction

Schrödinger's cat sums up one of the most striking and counter-intuitive features of quantum systems that is the ability to exist in coherent superpositions of states, which, in the classical world, would mutually exclude each other. While the conceptual ambiguities arising due to this phenomenon have been highly debated in the early days of quantum mechanics, during the last decades, it has been pointed out that quantum coherence might serve as valuable resource, especially for information processing. Among the first suggestions in this direction were the Brassard-Bennett protocol and the Deutsch-Jozsa algorithm promising respectively intrinsically eavesdrop-secure communication and an exponential speedup of computation by exploiting the quantum superposition principle [1]. Although these schemes are of little practical use so far, they reveal the enormous potential of quantum technologies, which nowadays becomes all the more significant due to recent experiments showing the accessibility of quantum effects even under ambient conditions [2–4].

Information thermodynamics [5–8] provides another, yet much less explored, area of research, which might benefit from the utilization of quantum coherence. The development of this field was originally triggered by Maxwell's famous thought experiment challenging the second law by invoking a small intelligent being, which is

able to separate the molecules of a gas in thermal equilibrium according to their velocity, thus establishing a spontaneous temperature gradient [9]. Building on Maxwell's idea, Szilárd invented a microscopic engine consisting of a single molecule confined in a container, which is in contact with a thermal reservoir of constant temperature [10]. An external agent might operate this setup by first dividing the container in two chambers, second, detecting the position of the molecule and, third, adiabatically expanding the chamber the molecule was found in, thus extracting work from a single heat bath. Half a century after its discovery, this apparent contradiction with the second law was resolved by Bennett [11], who showed that, due to Landauer's principle [12], the reduction of entropy associated with the measurement in the second step must be eventually compensated when the external agent discards the gathered information from its memory, which can not be inexhaustible. Hence, effectively, the information acquired during the measurement is converted into work. Meanwhile a fairly complete and experimentally confirmed theoretical framework exists [5, 13, 14] at least for classical systems, which, on a general level, provides precise extensions of the second law accounting for information as a physical quantity thus relating it to traditional thermodynamic variables such as entropy and work.

In the quantum realm, additional intricacies arise, which are not yet fully explored [15–30]. As a consequence of the superposition principle even the Hilbert space of a simple two-level system (TLS) contains infinitely many orthonormal pairs of realizable states, each of which is associated with a specific observable, which, in principle, might be measured. Moreover, according to the projection postulate, a measurement will typically alter the state of the system and thereby its mean energy. Therefore, in strong contrast to the classical case, a projective quantum measurement is not only accompanied by a decrease in entropy but also by an intrinsic change in internal energy, which must be taken into account for thermodynamic considerations. Jacobs argued that this energetic cost should be attributed to the external observer and derived an inequality, which incorporates it in an upper bound on the work extractable from a quantum system in thermal equilibrium after a single measurement [31]. Here, we go one step further by relaxing the assumption on the initial state and allowing multiple measurements in finite intervals. Using a simple argument based on the first and the second law, we show that Jacobs' bound holds whenever the probability to obtain a certain outcome does not change from one measurement to the next. This result, in particular, implies a bound on the average work delivered by information driven quantum engines operating periodically and in finite time. Moreover, it provides a natural definition for the efficiency of such machines.

One of the first specific, fully quantum mechanical models for a measurement controlled device was proposed by Lloyd [32]. In the spirit of Szilárd's pioneering work, he considers a single spin- $\frac{1}{2}$ system in contact with a thermal heat bath. An external controller can extract work in form of photons from this system by measuring the energy of the spin and applying a π -pulse at the Larmor frequency if the excited state is detected. After the spin-flip, or, if initially the ground state was observed, the system is allowed to return to thermal equilibrium, before the procedure repeats. Lloyd demonstrates that his engine can completely convert the information acquired by the measurement into work. Furthermore, he argues that the efficiency of this process will inevitably decrease, due to decoherence effects, if any observable different from energy is used to determine the state of the system. However, his reasoning strongly relies on the assumption that the spin has relaxed to thermal equilibrium before any measurement, which, in fact, would require an infinite waiting time.

In this work, by generalizing the setup described above, we show that triggering a laser pulse by measuring an observable that does not commute with the Hamiltonian of the system can enhance the efficiency if the model is operated in finite time. Specifically, we investigate a quantum-optical TLS, whose relaxation dynamics is modeled using a quantum master equation. After a projective von-Neumann measurement, the system is assumed to be detached from the heat bath such that its time evolution during the laser pulse is governed by a time-dependent Schrödinger equation. We note that such a separation of system and environment has recently been argued to be realistic in the context of quantum heat engines [33, 34]. For our model, we analytically calculate the time-dependent density matrix characterizing the system in the cyclic operation mode and numerically determine the optimal observable to control the feedback protocol as a function of the relaxation time and the spacing of the energy levels. Our findings show that exploiting quantum coherences can enhance the efficiency of information engines beyond classically achievable values.

The paper is structured as follows. As our first main result, we derive a new bound on the average work output of quantum information engines in section 2. In section 3, we introduce a specific model for such a machine and solve its dynamics. Section 4 is devoted to the optimization of its efficiency. We conclude in section 5.

2. Bound on work for cyclic quantum information engines

We begin this section by introducing a general scheme for a cyclic quantum information engine. To this end, we consider a finite quantum system with Hamiltonian H , which is in contact with a heat bath of temperature T and whose density matrix is initially given by ρ_{ini} . This setup is now operated by an external agent in two steps. First, an instantaneous projective measurement of the observable A is carried out, which yields the outcome a_m and leaves the system in the state

$$\rho_m \equiv |\psi_m\rangle\langle\psi_m| \quad (1)$$

with probability

$$p_m^{(0)} \equiv \langle\psi_m|\rho_{\text{ini}}|\psi_m\rangle. \quad (2)$$

Here, the a_m are the eigenvalues of A , which we assume to be non-degenerate, and $|\psi_m\rangle$ denotes the normalized eigenvector of A corresponding to a_m . Second, to convert the acquired information into useful work, a control operation is applied to the system, which is conditioned on the result of the preceding measurement and leads to the evolved density matrix

$$\tilde{\rho}_m \equiv \mathcal{V}_m[\rho_m], \quad (3)$$

where \mathcal{V}_m , in principle, can be any positive, trace preserving map [1]. Practically, such an operation can be realized by intermediately manipulating the Hamiltonian of the system or its coupling to the heat bath over a certain time interval.

The agent now iterates the sequence of steps one and two, where, in the i^{th} operation cycle, the measurement outcome $a_{m'}$ is obtained with probability $p_{m'}^{(i)}$. It is readily seen that these quantities fulfill the recursion relation

$$p_{m'}^{(i)} = \sum_m p[m'|m] p_m^{(i-1)} \quad (4)$$

with the conditional probability

$$p[m'|m] = \langle\psi_{m'}|\tilde{\rho}_m|\psi_{m'}\rangle = \langle\psi_{m'}|\mathcal{V}_m[|\psi_m\rangle\langle\psi_m|]|\psi_{m'}\rangle, \quad (5)$$

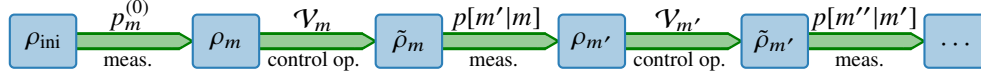


Figure 1. Flow chart illustration of the operation principle of a general quantum information engine. Alternately, the observable A is measured and the control operation \mathcal{V}_m conditioned on the measurement outcome a_m is applied to the system. Symbols are explained in the main text.

since the initial density matrix of each cycle is the result of the control operation applied in the foregoing one, as shown in figure 1. Moreover, since the transition probability (5) does not depend on the cycle index i but rather is fully determined by the control operation \mathcal{V}_m and the observable A , after sufficiently many iterations a stationary distribution $q_m \equiv \lim_{i \rightarrow \infty} p_m^{(i)}$ satisfying

$$q_{m'} = \sum_m p[m'|m] q_m \quad (6)$$

will be approached. Once this steady state is reached, the system works as a periodic information engine.

For a thermodynamic analysis of the scheme outlined above, we have to calculate the changes in internal energy $E[\rho] \equiv \text{tr}\{H\rho\}$ and entropy $S_{\text{sys}}[\rho] \equiv -k_B \text{tr}\{\rho \ln \rho\}$ of the system associated with the steps one and two, where k_B is Boltzmann's constant. Considering an operation cycle with initial density matrix $\tilde{\rho}_m$ and measurement outcome $a_{m'}$, we find

$$\Delta E^{\text{meas}}(m', m) = E[\rho_{m'}] - E[\tilde{\rho}_m] = \langle \psi_{m'} | H | \psi_{m'} \rangle - \text{tr}\{H \tilde{\rho}_m\}, \quad (7)$$

$$\Delta S_{\text{sys}}^{\text{meas}}(m', m) = S_{\text{sys}}[\rho_{m'}] - S_{\text{sys}}[\tilde{\rho}_m] = k_B \text{tr}\{\tilde{\rho}_m \ln \tilde{\rho}_m\} \quad (8)$$

for the measurement and

$$\Delta E^{\text{con}}(m') = E[\tilde{\rho}_{m'}] - E[\rho_{m'}] = \text{tr}\{H \tilde{\rho}_{m'}\} - \langle \psi_{m'} | H | \psi_{m'} \rangle, \quad (9)$$

$$\Delta S_{\text{sys}}^{\text{con}}(m') = S_{\text{sys}}[\tilde{\rho}_{m'}] - S_{\text{sys}}[\rho_{m'}] = -k_B \text{tr}\{\tilde{\rho}_{m'} \ln \tilde{\rho}_{m'}\} \quad (10)$$

for the control operation, where we used $S_{\text{sys}}[\rho_{m'}] = 0$ due to $\rho_{m'}$ representing a pure state. Since the total entropy production during the control step

$$\Delta S_{\text{tot}}^{\text{con}}(m') = \Delta S_{\text{sys}}^{\text{con}}(m') + \Delta S_{\text{bath}}^{\text{con}}(m') \geq 0 \quad (11)$$

must be nonnegative by virtue of the second law, it follows that the change in entropy of the heat bath $\Delta S_{\text{bath}}^{\text{con}}(m')$ is bounded from below by $-\Delta S_{\text{sys}}^{\text{con}}(m')$ and thus the heat taken up by the system $Q(m') = -T \Delta S_{\text{bath}}^{\text{con}}(m')$ during the control operation $\mathcal{V}_{m'}$ is bounded from above by $T \Delta S_{\text{sys}}^{\text{con}}(m')$. Consequently, the first law

$$\Delta E^{\text{con}}(m') = Q(m') - W(m') \quad (12)$$

implies the bound

$$W(m') \leq T \Delta S_{\text{sys}}^{\text{con}}(m') - \Delta E^{\text{con}}(m') \quad (13)$$

on the work $W(m')$ the agent can extract from the system using the operation $\mathcal{V}_{m'}$.

Since the measurement outcome a_m occurs with probability q_m in the steady state, (13) yields the bound

$$\langle W \rangle \equiv \sum_m q_m W(m) \quad (14)$$

$$\leq \sum_m q_m \left(-k_B T \text{tr} \{ \tilde{\rho}_m \ln \tilde{\rho}_m \} - \text{tr} \{ H \tilde{\rho}_m \} + \langle \psi_m | H | \psi_m \rangle \right) \quad (15)$$

on the average work extracted per operation cycle. Furthermore, the average energetic cost and entropy reduction per cycle associated with the measurement read

$$\langle \Delta E^{\text{meas}} \rangle \equiv \sum_{m', m} p(m', m) \Delta E^{\text{meas}}(m', m) \quad (16)$$

and

$$\langle \Delta S_{\text{sys}}^{\text{meas}} \rangle \equiv \sum_{m', m} p(m', m) \Delta S_{\text{sys}}^{\text{meas}}(m', m), \quad (17)$$

respectively. Here, $p(m', m) \equiv p[m'|m]q_m$ is the probability to measure $a_{m'}$ and a_m in two consecutive operation cycles. Inserting (7) and (8) into (16) and (17) and using the steady state condition (6) as well as the sum rule

$$\sum_{m'} p[m'|m] = 1 \quad (18)$$

expressing probability conservation yields

$$\langle \Delta E^{\text{meas}} \rangle = \sum_m q_m (\langle \psi_m | H | \psi_m \rangle - \text{tr} \{ H \tilde{\rho}_m \}), \quad (19)$$

$$\langle \Delta S_{\text{sys}}^{\text{meas}} \rangle = \sum_m q_m k_B \text{tr} \{ \tilde{\rho}_m \ln \tilde{\rho}_m \}. \quad (20)$$

By comparing (19) and (20) with (15), we obtain the bound

$$\langle W \rangle \leq -T \langle \Delta S_{\text{sys}}^{\text{meas}} \rangle + \langle \Delta E^{\text{meas}} \rangle. \quad (21)$$

This inequality, which constitutes our first main result, provides a universal upper bound on the average work extractable per operation cycle in terms of quantities that are related to the measurement process only. It generalizes similar results obtained in [31, 35–37] for single stroke operations. Following the arguments of Jacobs [31], we consider the energetic cost of the measurement $\langle \Delta E^{\text{meas}} \rangle$ as work input provided by the measurement apparatus and thus infer from (21) the natural definition

$$\eta \equiv \frac{\langle W \rangle}{\langle \Delta E^{\text{meas}} \rangle - T \langle \Delta S_{\text{sys}}^{\text{meas}} \rangle} \leq 1 \quad (22)$$

for the efficiency, at which information is converted to work in cyclic quantum engines. We note that, while $-\langle \Delta S_{\text{sys}}^{\text{meas}} \rangle$ is readily seen to be always nonnegative, in contrast to the setup considered in [31], $\langle \Delta E^{\text{meas}} \rangle$ can, in principle, become negative, since, for finite cycle times, the system will typically not be in thermal equilibrium before the measurement is performed. Moreover, the quantity $\langle \Delta E^{\text{meas}} \rangle$ is of pure quantum origin and vanishes in the quasi-classical situation, where the observable A commutes with the Hamiltonian of the system H .

3. Quantum optical model

As an application of the general theory discussed so far, we propose a generalization of a paradigmatic model for a quantum information engine originally invented by Lloyd [32] and analyze its thermodynamic properties. Specifically, we consider an optical TLS with Hamiltonian

$$H = \frac{\hbar\omega_0}{2} (|e\rangle\langle e| - |g\rangle\langle g|), \quad (23)$$

where $\hbar\omega_0 > 0$ is the energetic spacing between the ground state $|g\rangle$ and the excited state $|e\rangle$. The external agent measures the observable $A(\theta)$ ($0 \leq \theta \leq \pi/2$) with eigenvalues $a_{\pm} = \pm 1$ and corresponding eigenvectors

$$|\psi_+(\theta)\rangle \equiv |\psi_+\rangle \equiv \cos \frac{\theta}{2} |e\rangle + \sin \frac{\theta}{2} |g\rangle, \quad (24)$$

$$|\psi_-(\theta)\rangle \equiv |\psi_-\rangle \equiv \sin \frac{\theta}{2} |e\rangle - \cos \frac{\theta}{2} |g\rangle, \quad (25)$$

which reduce to the eigenstates of H for $\theta = 0$. In order to extract work in form of photons, after a measurement of the state $|\psi_+\rangle$, the system is detached from the heat bath and a coherent laser pulse on resonance is applied for an interval t_F . After a measurement of $|\psi_-\rangle$, the system is kept in contact with the thermal environment for a time t_R without any action of the agent to allow the absorption of additional heat before the next measurement is carried out.

For a quantitative description of this procedure, which is summarized in figure 2, we need to specify the control operations \mathcal{V}_{\pm} . During the interaction ($0 \leq \tau \leq t_F$) with the laser pulse, the density matrix $\rho(\tau)$ of the system evolves unitarily according to the Liouville-von Neumann equation

$$\partial_{\tau}\rho(\tau) = -\frac{i}{\hbar}[H_F(\tau), \rho(\tau)] \equiv \mathcal{L}_+(\tau)\rho(\tau), \quad (26)$$

where, on the semiclassical level and within the rotating wave approximation, the time-dependent Hamiltonian is given by

$$H_F(\tau) \equiv H + \frac{\hbar\Omega}{2} \left(e^{-i(\omega_0\tau-\phi)} |e\rangle\langle g| + e^{i(\omega_0\tau-\phi)} |g\rangle\langle e| \right) \quad (27)$$

with real Rabi frequency $\Omega > 0$ and $0 \leq \phi < 2\pi$ being the phase of the dipole matrix element [38]. To describe the interaction of the TLS with the heat bath during $0 \leq \tau \leq t_R$, we use the quantum optical master equation [39]

$$\begin{aligned} \partial_{\tau}\rho(\tau) = & -\frac{i}{\hbar}[H, \rho(\tau)] + \gamma N \left(L\rho(\tau)L^{\dagger} - \frac{1}{2}L^{\dagger}L\rho(\tau) - \frac{1}{2}\rho(\tau)L^{\dagger}L \right) \\ & + \gamma(N+1) \left(L^{\dagger}\rho(\tau)L - \frac{1}{2}LL^{\dagger}\rho(\tau) - \frac{1}{2}\rho(\tau)LL^{\dagger} \right) \\ \equiv & \mathcal{L}_-\rho(\tau), \end{aligned} \quad (28)$$

where $L \equiv |e\rangle\langle g|$ is a Lindblad operator, $N \equiv 1/(\exp[\hbar\omega_0/(k_B T)] - 1)$ denotes the Planck distribution evaluated for the level spacing $\hbar\omega_0$ and $\gamma > 0$ is a damping rate quantifying the coupling strength between the TLS and the thermal reservoir. This time evolution equation, which is of Lindblad form and therefore preserves trace and positivity of the density matrix, can be derived from a microscopic model in the weak

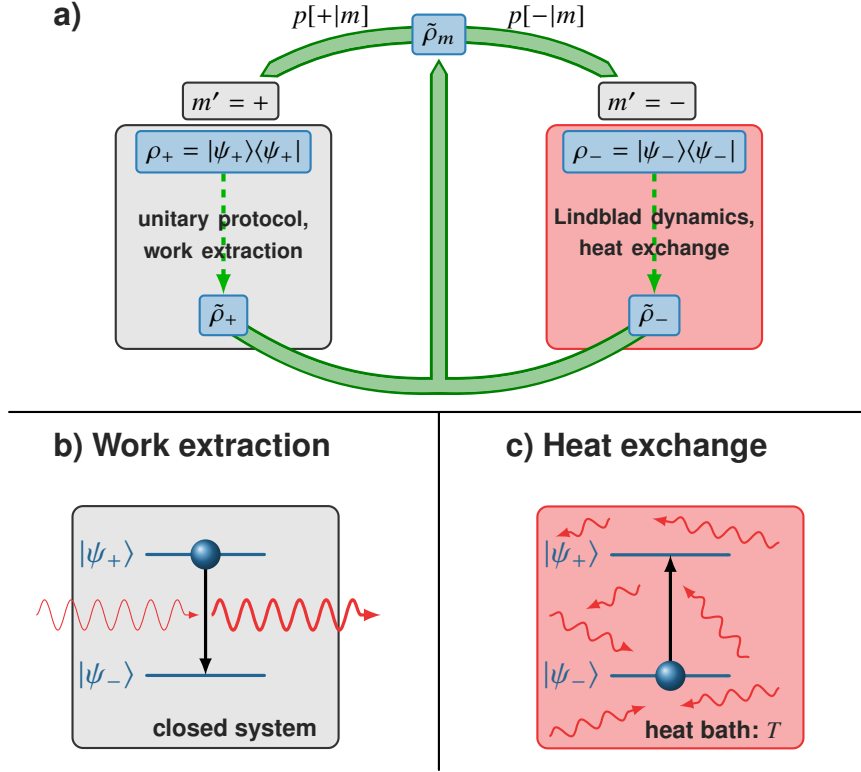


Figure 2. Scheme of the quantum optical TLS as information engine (a). At the beginning of each operation cycle, the state of the system is inferred by the agent via a measurement of the observable $A(\theta)$. If the outcome of this measurement is a_+ , the internal energy of the TLS is used to coherently amplify an externally generated laser pulse (b). If the outcome is a_- , the susceptibility of the system for further energy uptake is exploited to extract heat from the environment (c). In any case, the density matrix at the end of the operation cycle serves as initial state for the subsequent one. For further explanations of the symbols, see main text.

coupling limit, where the role of the heat bath is played by the thermal radiation field, for details, see [39]. Such master equations are a well established method for the description of open quantum systems, which has previously lead to substantial insights in the context of quantum heat engines, see for example [40, 41]. In terms of the super operators \mathcal{L}_\pm , the control operations admit the formal representations

$$\mathcal{V}_+[\rho] = \overrightarrow{\mathcal{T}} e^{\int_0^{t_F} d\tau \mathcal{L}_+(\tau)} \rho \quad \text{and} \quad \mathcal{V}_-[\rho] = e^{\mathcal{L}_- t_R} \rho, \quad (29)$$

where $\overrightarrow{\mathcal{T}}$ indicates time ordering. Solving the equations (26) and (28) for a general initial condition yields the explicit expressions [38]

$$\begin{aligned} \mathcal{V}_+[\rho] &= U \rho U^\dagger \quad \text{with} \\ U &= \cos \frac{\Omega t_F}{2} \left(e^{i\omega_0 t_F/2} |g\rangle\langle g| + e^{-i\omega_0 t_F/2} |e\rangle\langle e| \right) \\ &\quad - i \sin \frac{\Omega t_F}{2} \left(e^{i(\omega_0 t_F/2 - \phi)} L^\dagger + e^{-i(\omega_0 t_F/2 - \phi)} L \right) \end{aligned} \quad (30)$$

and [42]

$$\begin{aligned}\mathcal{V}_-[\rho] = e^{\mathcal{L}-t_R}\rho = & \frac{1}{4} \left(1 + e^{-\Gamma t_R} + 2e^{-\Gamma t_R/2} \cos \omega_0 t_R \right) \rho \\ & + \frac{1}{4} \left(1 + e^{-\Gamma t_R} - 2e^{-\Gamma t_R/2} \cos \omega_0 t_R \right) L_0 \rho L_0 \\ & - \frac{1}{4} \left(\frac{\gamma}{\Gamma} (1 - e^{-\Gamma t_R}) - 2ie^{-\Gamma t_R/2} \sin \omega_0 t_R \right) \rho L_0 \\ & - \frac{1}{4} \left(\frac{\gamma}{\Gamma} (1 - e^{-\Gamma t_R}) + 2ie^{-\Gamma t_R/2} \sin \omega_0 t_R \right) L_0 \rho \\ & + (1 - e^{-\Gamma t_R}) \left(\frac{\gamma(N+1)}{\Gamma} L^\dagger \rho L + \frac{\gamma N}{\Gamma} L \rho L^\dagger \right),\end{aligned}\quad (31)$$

where we introduced the abbreviations $L_0 \equiv |e\rangle\langle e| - |g\rangle\langle g|$ and $\Gamma \equiv \gamma(2N+1)$.

The work extracted within an operation cycle with measurement outcome a_+ can be determined from the first law

$$W(+) = -\Delta E^{\text{con}}(+) = E[\rho_+] - E[\tilde{\rho}_+] = E[\rho_+] - E[\mathcal{V}_+[\rho_+]], \quad (32)$$

since the TLS is decoupled from the environment and thus no heat is exchanged during the control operation \mathcal{V}_+ . Inserting $\rho_+ = |\psi_+\rangle\langle\psi_+|$ and (30) into (32) gives

$$W(+) = \frac{\hbar\omega_0}{2} \left(\cos \theta (1 - \cos \Omega t_F) - \sin \theta \sin \phi \sin \Omega t_F \right). \quad (33)$$

To keep the subsequent analysis as simple as possible, from here onwards, we fix the pulse duration t_F and the dipole phase ϕ such that (33) assumes the maximal value

$$W(+) = \hbar\omega_0 \cos^2 \frac{\theta}{2} \quad (34)$$

with respect to these parameters, i.e., we put

$$t_F = \frac{\theta + \pi}{\Omega} \quad \text{and} \quad \phi = \frac{\pi}{2}. \quad (35)$$

This choice ensures that the TLS ends up in the ground state after the laser pulse, i.e., $\tilde{\rho}_+ = |g\rangle\langle g|$. Furthermore, it leads to the fairly simple expressions

$$p[+|+] = \sin^2 \frac{\theta}{2} \quad (36)$$

and

$$p[+|-] = \frac{1}{2} \left(1 - \frac{\cos \theta}{2N+1} - e^{-\Gamma t_R/2} \sin^2 \theta \cos \omega_0 t_R - e^{-\Gamma t_R} \cos \theta \left[\cos \theta - \frac{1}{2N+1} \right] \right) \quad (37)$$

for the conditional probabilities defined in (5). Since $p[-|+]$ and $p[-|-]$ are determined by the sum rules (18), the steady state probabilities q_\pm can now be obtained from the fixed point condition (6). Specifically, we find

$$q_+ = \frac{p[+|-]}{1 - p[+|+] + p[+|-]} = 1 - q_-. \quad (38)$$

We are now ready to calculate the quantities entering the efficiency (22). First, the average work per cycle reads

$$\langle W \rangle = q_+ W(+), \quad (39)$$

since no contribution arises from operation cycles with measurement outcome a_- . Second, the average energy spent on the measurement (19) becomes

$$\begin{aligned}\langle \Delta E^{\text{meas}} \rangle &= \sum_{m=\pm} q_m \left(E[\rho_m] - E[\mathcal{V}_m[\rho_m]] \right) \\ &= \langle W \rangle + q_- \frac{\hbar\omega_0}{2} (1 - e^{-\Gamma t_R}) \left(\frac{1}{2N+1} - \cos \theta \right)\end{aligned}\quad (40)$$

upon using $\rho_{\pm} = |\psi_{\pm}\rangle\langle\psi_{\pm}|$ and the expressions (30) and (31) for the control operations. Third, since $\tilde{\rho}_+$ represents a pure state due to the control operation \mathcal{V}_+ being unitary, the average entropy reduction in the system associated with the measurement (20) arises only from cycles with initial state $\tilde{\rho}_-$. After some algebra again using (31), we thus obtain

$$\begin{aligned}\langle \Delta S_{\text{sys}}^{\text{meas}} \rangle &= q_- k_B \text{tr} \{ \mathcal{V}_-[\rho_-] \ln \mathcal{V}_-[\rho_-] \} \\ &= \frac{q_-}{2} k_B \left(\ln D + \sqrt{1-4D} \ln \left(\frac{1 + \sqrt{1-4D}}{1 - \sqrt{1-4D}} \right) \right) \quad \text{with}\end{aligned}\quad (41)$$

$$D \equiv \frac{1}{4} \left(1 - \left(e^{-\Gamma t_R} \cos \theta - \frac{e^{-\Gamma t_R} - 1}{2N+1} \right)^2 - e^{-\Gamma t_R} \sin^2 \theta \right). \quad (42)$$

Using the expressions (39)-(42), the efficiency η of this quantum optical information engine can be evaluated for any complete set of parameters comprising the level spacing $\hbar\omega_0$, the temperature of the heat bath T , the angle θ , the damping rate γ and the relaxation time t_R .

4. Quasi-classical vs. coherence-enhanced regime

In this section, we focus on the question whether coherences, i.e., a choice $\theta \neq 0$ for the basis, in which the measurement is performed, can enhance the efficiency η . In order to reduce the number of free parameters, we choose from now on the temperature such that

$$x \equiv \hbar\omega_0/k_B T = \ln 2, \quad (43)$$

leading to $N = 1$. We then find by inspection that η depends only on θ and the two dimensionless parameters γt_R and ω_0/γ . A numerical optimization procedure yields the maximal efficiency η^* and optimal angle θ^* , which are both shown in upper panels of figure 3. These plots exhibit two qualitatively different regimes separated by $\gamma t_R \simeq 1.5$.

First, for $\gamma t_R \gtrsim 1.5$, we recover the quasi-classical regime originally considered by Lloyd [32], within which the TLS can relax to thermal equilibrium in each operation cycle with measurement outcome a_- . As argued in [32], the largest efficiency can then be obtained for $\theta = 0$. Consistently, we observe that θ^* is effectively zero in this regime and, independent of ω_0/γ , the optimal efficiency settles at the constant value

$$\eta^* \approx \frac{x}{(1 + e^x) \ln(1 + e^x) - x e^x} \simeq 0.36. \quad (44)$$

Second, in the coherent regime $\gamma t_R \lesssim 1.5$, we find a characteristic oscillatory pattern, which can be traced back to the structure of the conditional probability (37).

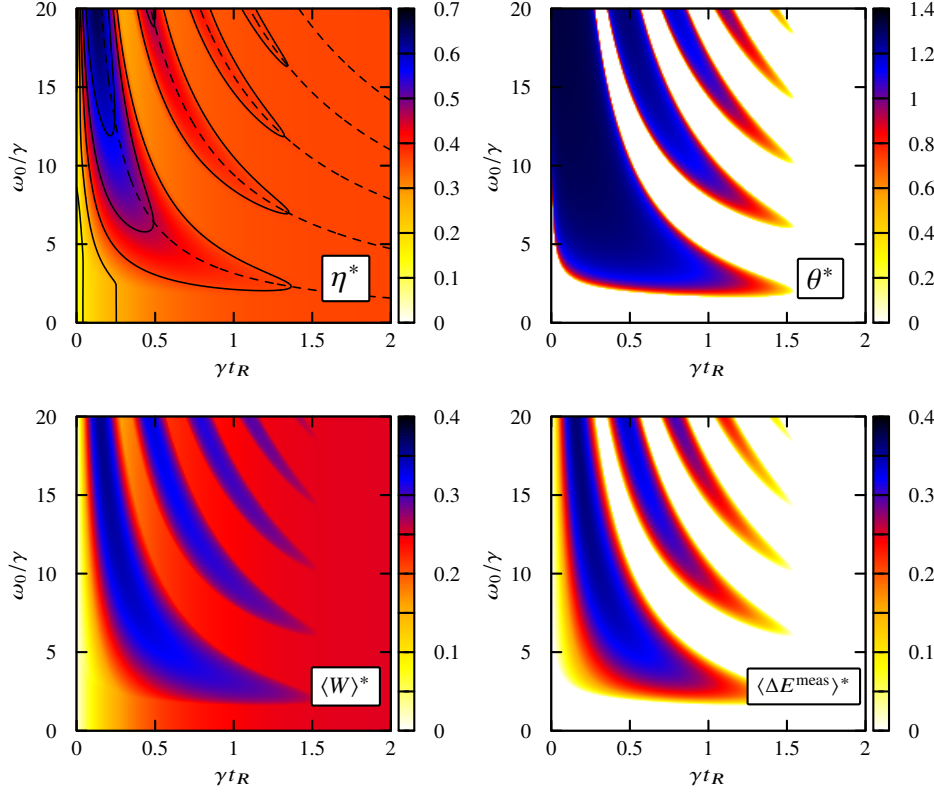


Figure 3. Benchmark parameters for the performance of the optimized quantum optical TLS information engine as functions of the dimensionless parameters γt_R and ω_0/γ . The upper panel shows the maximum efficiency η^* on the left and the corresponding optimal angle θ^* on the right. Along the dashed lines, the condition (46) is fulfilled. The solid lines, which have a spacing of 0.1 and constant offset 0.1625, were introduced for graphical purposes. In the lower panel, the work output $\langle W \rangle^*$ (left) and the average energy input required per measurement $\langle \Delta E^{\text{meas}} \rangle^*$ (right) is plotted in units of $\hbar\omega_0$.

The crucial role of this quantity, which is, in fact, the only ingredient of the efficiency depending on ω_0/γ , can be explained by the following argument. If the TLS is found in the state $|\psi_+\rangle$ after being in contact with the thermal environment for the time t_R , the heat absorbed during this period together with the energy invested for the measurement can be converted into work by applying a laser pulse. A measurement indicating the state $|\psi_-\rangle$, however, leads to another relaxation cycle, within which no work can be extracted and the previously gained information is inevitably wasted. Consequently, the efficiency, at which acquired information is converted into work, can be expected to increase as the frequency of such idle cycles decreases. For $\gamma t_R \lesssim 1.5$,

the corresponding probability

$$\begin{aligned} p[-|-] &= 1 - p[+|-] \\ &= \frac{1}{2} \left(1 + \frac{\cos \theta}{3} + e^{-3\gamma t_R/2} \sin^2 \theta \cos \omega_0 t_R + e^{-3\gamma t_R} \cos \theta \left[\cos \theta - \frac{1}{3} \right] \right) \end{aligned} \quad (45)$$

can be substantially reduced by the contribution proportional to $e^{-3\gamma t_R/2}$, which arises solely due to quantum coherences and vanishes for $\theta = 0$. This effect becomes most pronounced for $\theta = \pi/2$ and

$$\omega_0 t_R = (\omega_0/\gamma) \cdot (\gamma t_R) = (2n + 1)\pi, \quad (n = 0, 1, 2, \dots). \quad (46)$$

Accordingly, the hyperbolas (46) are in good agreement with the local maxima of the efficiency η^* in the $(\gamma t_R, \omega_0/\gamma)$ -plane and θ^* comes close to $\pi/2$ in their vicinity. The deviations from this pattern for $\gamma t_R \lesssim 0.5$ can be explained by the remaining terms in (45), which come with a prefactor $e^{-3\gamma t_R}$ and thus give a non-negligible contribution only in these regions. Most importantly, in this regime, we find as our second main result that the efficiency is enhanced by exploiting coherences. Specifically, it can overcome the quasi-classical value (44) and even approach its upper bound 1 in the limit $\omega_0/\gamma \rightarrow \infty$.

The average work output for the optimal angle θ^* , $\langle W \rangle^*$, is plotted in the lower panel of figure 3. Clearly, this quantity features the same characteristic dependence on γt_R and ω_0/γ as the maximized efficiency η^* . Like η^* , the average work $\langle W \rangle^*$ exceeds its quasi-classical limit

$$\lim_{\gamma t_R \rightarrow \infty} \langle W \rangle^* = \hbar \omega_0 / 4 \quad (47)$$

in the coherence-enhanced regime and becomes maximal in the same range of parameters like η^* , i.e., in the vicinity of the hyperbolas (46).

Finally, we consider the average energetic cost per measurement $\langle \Delta E^{\text{meas}} \rangle^*$. This additional input is inevitably necessary for the exploitation of quantum coherence and therefore becomes non-negligible whenever θ^* significantly deviates from 0, hence, in particular, in the regions of the parameter space, where our numerical procedure reveals η^* to be large. Consequently, in the range of high efficiencies, the input of the device is mainly delivered by the measurement apparatus rather than the heat bath. This result underlines the crucial role of the measurement process in the quantum realm, which, besides delivering information, can alter the state of the system and thus bears the character of an additional control operation.

5. Conclusion

In this paper, we have derived a universal upper bound on the average work output delivered in finite time by cyclically operating quantum information engines, which takes into account the energetic cost intrinsically associated with quantum measurements. This bound provides a benchmark for the performance of quantum mechanical information-to-energy converters, which, in contrast to their classical counterparts, see for example [43–51], can exploit the superposition principle and thus might be able to overcome classical limitations.

We have explicitly investigated the benefit of quantum coherences in the second part of the paper by considering a specific model consisting of a quantum optical TLS, which, conditioned on the outcome of a projective measurement, is repeatedly

either coupled to a heat bath or used to amplify a coherent laser pulse. In the regime of long relaxation times, this setup corresponds to a model originally proposed by Lloyd, whose properties are reproduced qualitatively in our analysis within this limit. We emphasize, however, that the definition of efficiency used in [32] is different from ours, since it explicitly refers to Landauer's principle by invoking the minimal heat that must be dissipated in a second heat bath of different temperature to achieve the entropy production necessary to reset the memory of the external agent. Viewed in this way, the model acts effectively as a heat engine, whose efficiency is bounded by the Carnot value. In our approach, we consider the system as an information engine and define its efficiency in terms of quantities directly associated with the system and the measurement process, leaving aside how the agent eventually erases the gathered information.

In the coherent regime, which is characterized by short cycle times, we find that utilizing a non-classical observable $A(\theta)$, whose eigenstates are coherent superpositions of the energy eigenstates, can enhance the performance significantly. Remarkably, it turns out that both, efficiency and average work output per cycle, can be substantially increased if the relaxation time is properly adjusted to the level spacing. Since, in the corresponding regions of the parameter space, the optimal angle θ^* strongly deviates from the quasi-classical value 0 and even comes close to $\pi/2$, the device is then mainly supplied by the measurement apparatus rather than the heat bath. In fact, Jacobs argued that, for thermodynamical consistency, this type of energy input must be considered as work rather than heat [31]. It should, however, also be clearly distinguished from the work output extracted by external control operations. Further clarification of the role of the measurement process in this context, using e.g. a scheme proposed in [52], constitutes an important and challenging subject for future research.

References

- [1] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information: 10th Anniversary Edition*. Cambridge University Press, 2010.
- [2] F. Dolde, I. Jakobi, B. Naydenov, N. Zhao, S. Pezzagna, C. Trautmann, J. Meijer, P. Neumann, F. Jelezko, and J. Wrachtrup, "Room-temperature entanglement between single defect spins in diamond," *Nature Phys.* **9** (2013) 139–143.
- [3] G. Waldherr, Y. Wang, S. Zaiser, M. Jamali, T. Schulte-Herbruggen, H. Abe, T. Ohshima, J. Isoya, J. F. Du, P. Neumann, and J. Wrachtrup, "Quantum error correction in a solid-state hybrid spin register," *Nature* **506** (2014) 204–207.
- [4] D. G. England, G. Fisher, Kent A. J.-P. W. MacLean, P. J. Bustard, R. Lausten, K. J. Resch, and B. J. Sussman, "Storage and Retrieval of THz-Bandwidth Single Photons Using a Room-Temperature Diamond Quantum Memory," *Phys. Rev. Lett.* **114** (2015) 053602.
- [5] J. M. R. Parrondo, J. M. Horowitz, and T. Sagawa, "Thermodynamics of information," *Nature Phys.* **11** (2015) 131–139.
- [6] S. Deffner and C. Jarzynski, "Information Processing and the Second Law of Thermodynamics: An Inclusive, Hamiltonian Approach," *Phys. Rev. X* **3** (2013) 041003.
- [7] A. C. Barato and U. Seifert, "Unifying Three Perspectives on Information Processing in Stochastic Thermodynamics," *Phys. Rev. Lett.* **112** (2014) 090601.
- [8] J. M. Horowitz and M. Esposito, "Thermodynamics with Continuous Information flow," *Phys. Rev. X* **4** (2014) 031015.
- [9] H. S. Leff and A. F. Rex, eds., *Maxwell's Demon 2: Entropy, Classical and Quantum Information, Computing*. IOP, Bristol and Philadelphia, 2003.
- [10] L. Szilard, "Über die Entropieverminderung in einem thermodynamischen System bei Eingriffen intelligenter Wesen," *Z. Phys.* **53** (1929) 840 – 856.
- [11] C. Bennett, "The thermodynamics of computation -a review," *Int. J. Theor. Phys.* **21** (1982) 905–940.

- [12] R. Landauer, “Irreversibility and heat generation in the computing process,” *IBM J. Res. Dev.* **5** (1961) 183–191.
- [13] S. Toyabe, T. Sagawa, M. Ueda, E. Muneyuki, and M. Sano, “Experimental demonstration of information-to-energy conversion and validation of the generalized Jarzynski equality,” *Nature Phys.* **6** (2010) 988.
- [14] A. Bérut, A. Arakelyan, A. Petrosyan, S. Ciliberto, R. Dillenschneider, and E. Lutz, “Experimental verification of Landauer’s principle linking information and thermodynamics,” *Nature* **483** (2012) 187–189.
- [15] W. H. Zurek, “Quantum discord and Maxwell’s demons,” *Phys. Rev. A* **67** (2003) 012320.
- [16] M. O. Scully, M. S. Zubairy, G. S. Agarwal, and H. Walther, “Extracting work from a single heat bath via vanishing quantum coherence,” *Science* **299** (2003) 862–864.
- [17] H. T. Quan, Y. D. Wang, Y.-x. Liu, C. P. Sun, and F. Nori, “Maxwell’s demon assisted thermodynamic cycle in superconducting quantum circuits,” *Phys. Rev. Lett.* **97** (2006) 180402.
- [18] A. E. Allahverdyan, R. S. Johal, and G. Mahler, “Work extremum principle: Structure and function of quantum heat engines,” *Phys. Rev. E* **77** (2008) 041118.
- [19] S. W. Kim, T. Sagawa, S. D. Liberato, and M. Ueda, “Quantum Szilard Engine,” *Phys. Rev. Lett.* **106** (2011) 070401.
- [20] K. Jacobs, “Quantum measurement and the first law of thermodynamics: The energy cost of measurement is the work value of the acquired information,” *Phys. Rev. E* **86** (2012) 040106.
- [21] D. Gelbwaser-Klimovsky, N. Erez, R. Alicki, and G. Kurizki, “Work extraction via quantum nondemolition measurements of qubits in cavities: Non-markovian effects,” *Phys. Rev. A* **88** (2013) 022112.
- [22] J. Yi and Y. W. Kim, “Nonequilibrium work and entropy production by quantum projective measurements,” *Phys. Rev. E* **88** (2013) 032105.
- [23] P. Strasberg, G. Schaller, T. Brandes, and M. Esposito, “Thermodynamics of quantum-jump-conditioned feedback control,” *Phys. Rev. E* **88** (2013) 062107.
- [24] J. J. Park, K.-H. Kim, T. Sagawa, and S. W. Kim, “Heat engine driven by purely quantum information,” *Phys. Rev. Lett.* **111** (2013) 230402.
- [25] K. Funo, Y. Watanabe, and M. Ueda, “Thermodynamic work gain from entanglement,” *Phys. Rev. A* **88** (2013) 052319.
- [26] J. M. Horowitz and K. Jacobs, “Quantum effects improve the energy efficiency of feedback control,” *Phys. Rev. E* **89** (2014) 042134.
- [27] S. Gasparinetti, P. Solinas, A. Braggio, and M. Sassetti, “Heat-exchange statistics in driven open quantum systems,” *New J. Phys.* **16** (2014) 115001.
- [28] M. F. Frenzel, D. Jennings, and T. Rudolph, “Reexamination of pure qubit work extraction,” *Phys. Rev. E* **90** (2014) 052136.
- [29] S. An, J.-N. Zhang, M. Um, D. Lv, Y. Lu, J. Zhang, Z.-Q. Yin, H. T. Quan, and K. Kim, “Experimental test of the quantum Jarzynski equality with a trapped-ion system,” *Nature Phys.* **11** (2015) 193–199.
- [30] J. Goold, M. Paternostro, and K. Modi, “Nonequilibrium Quantum Landauer Principle,” *Phys. Rev. Lett.* **114** (2015) 060602.
- [31] K. Jacobs, “Second law of thermodynamics and quantum feedback control: Maxwell’s demon with weak measurements,” *Phys. Rev. A* **80** (2009) 012322.
- [32] S. Lloyd, “Quantum-mechanical Maxwell’s demon,” *Phys. Rev. A* **56** (1997) 3374–3382.
- [33] O. Abah, J. Roßnagel, G. Jacob, S. Deffner, F. Schmidt-Kaler, K. Singer, and E. Lutz, “Single-Ion Heat Engine at Maximum Power,” *Phys. Rev. Lett.* **109** (2012) 203006.
- [34] J. Roßnagel, O. Abah, F. Schmidt-Kaler, K. Singer, and E. Lutz, “Nanoscale Heat Engine Beyond the Carnot Limit,” *Phys. Rev. Lett.* **112** (2014) 030602.
- [35] T. Sagawa and M. Ueda, “Second law of thermodynamics with discrete quantum feedback control,” *Phys. Rev. Lett.* **100** (2008) 080403.
- [36] H.-H. Hasegawa, J. Ishikawa, K. Takara, and D. Driebe, “Generalization of the second law for a nonequilibrium initial state,” *Phys. Lett. A* **374** (2010) 1001 – 1004.
- [37] N. Erez, “Thermodynamics of projective quantum measurements,” *Phys. Scr.* (2012) 014028.
- [38] M. O. Scully and M. S. Zubairy, *Quantum Optics*. Cambridge Univ. Press, 2008.
- [39] H. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*. OUP Oxford, 2007.
- [40] E. Geva and R. Kosloff, “Three-level quantum amplifier as a heat engine: A study in finite-time thermodynamics,” *Phys. Rev. E* **49** (1994) 3903–3918.
- [41] E. Geva, R. Kosloff, and J. L. Skinner, “On the relaxation of a two-level system driven by a strong electromagnetic field,” *J. Chem. Phys.* **102** (1995) 8541–8561.
- [42] H. Nakazato, Y. Hida, K. Yuasa, B. Militello, A. Napoli, and A. Messina, “Solution of the

- Lindblad equation in the Kraus representation,” *Phys. Rev. A* **74** (2006) 062113.
- [43] M. Bauer, D. Abreu, and U. Seifert, “Efficiency of a Brownian information machine,” *J. Phys. A* **45** (2012) 162001.
 - [44] M. Bauer, A. C. Barato, and U. Seifert, “Optimized finite-time information machine,” *J. Stat. Mech.* (2014) P09010.
 - [45] A. E. Allahverdyan, D. Janzing, and G. Mahler, “Thermodynamic efficiency of information and heat flow,” *J. Stat. Mech.* (2009) P09011.
 - [46] D. Abreu and U. Seifert, “Extracting work from a single heat bath through feedback,” *EPL* **94** (2011) 10001.
 - [47] J. M. Horowitz and J. M. R. Parrondo, “Designing optimal discrete-feedback thermodynamic engines,” *New J. Phys.* **13** (2011) 123019.
 - [48] D. Mandal and C. Jarzynski, “Work and information processing in a solvable model of Maxwell’s demon,” *Proc. Natl. Acad. Sci. U.S.A.* **109** (2012) 11641–11645.
 - [49] J. M. Horowitz, T. Sagawa, and J. M. R. Parrondo, “Imitating Chemical Motors with Optimal Information Motors,” *Phys. Rev. Lett.* **111** (2013) 010602.
 - [50] A. C. Barato and U. Seifert, “An autonomous and reversible Maxwell’s demon,” *EPL* **101** (2013) 60001.
 - [51] H. Sandberg, J.-C. Delvenne, N. J. Newton, and S. K. Mitter, “Maximum work extraction and implementation costs for nonequilibrium Maxwell’s demons,” *Phys. Rev. E* **90** (2014) 042119.
 - [52] A. E. Allahverdyan, R. Balian, and T. M. Nieuwenhuizen, “Understanding quantum measurement from the solution of dynamical models,” *Phys. Rep.* **525** (2013) 1 – 166.